

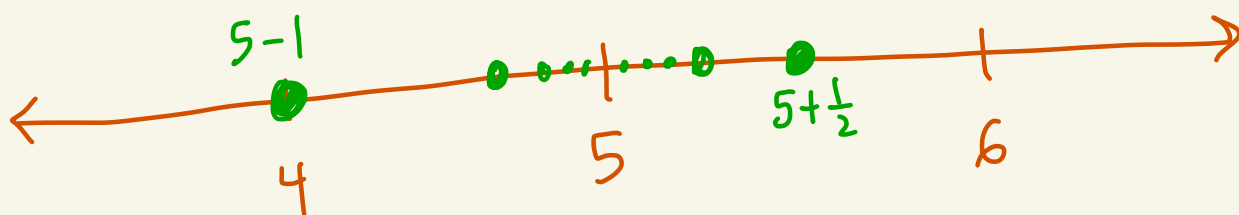
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Test 1 Solutions



①(a)

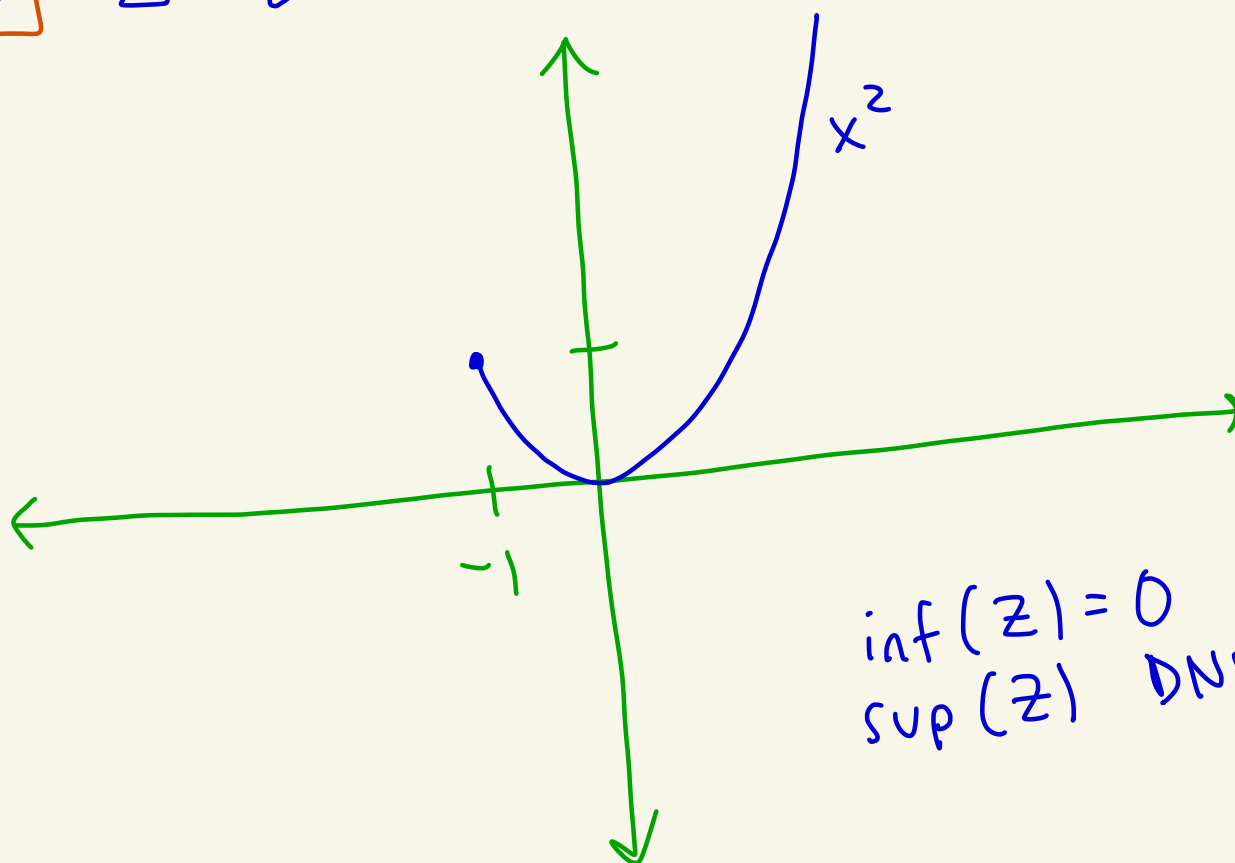
$$Y = \left\{ 5-1, 5+\frac{1}{2}, 5-\frac{1}{3}, 5+\frac{1}{4}, 5-\frac{1}{5}, \dots \right\}$$



$$\inf(Y) = 4$$

$$\sup(Y) = 5.5$$

①(b) $Z = \{x^2 \mid x \in \mathbb{R} \text{ with } -1 \leq x\}$



$$\inf(Z) = 0$$

$$\sup(Z) \text{ DNE}$$

(2) Let $\varepsilon > 0$.

Note that

$$\begin{aligned} \left| \frac{n}{2n+1} - \frac{1}{2} \right| &= \left| \frac{2n - 2n - 1}{2n+1} \right| \\ &= \left| \frac{-1}{2n+1} \right| = \frac{1}{2n+1} \end{aligned}$$

We have that $\frac{1}{2n+1} < \varepsilon$

$$\text{iff } \frac{1}{\varepsilon} < 2n+1$$

$$\text{iff } \frac{1}{2\varepsilon} - \frac{1}{2} < n.$$

Let $N > \frac{1}{2\varepsilon} - \frac{1}{2}$.

Then if $n \geq N > \frac{1}{2\varepsilon} - \frac{1}{2}$ we have that

$$\left| \frac{n}{2n+1} - \frac{1}{2} \right| = \frac{1}{2n+1} < \varepsilon.$$

Thus, $\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2}$



(A) HW 1 # 6(a)

(B) HW 1 # 7(a)

(C) HW 1 # 8(b)

(D) See class notes from 9/8 pg. 6.

(E) HW 2 # 4(b)

(F) HW 2 # 5
